Further Maths Revision Paper 1

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q1, 2 and 3)

1

Solve

$$\frac{4x+1}{x+2} \leqslant \frac{5}{x-3}, \qquad x \neq -2, x \neq 3$$

2

The tangent at a point P on the parabola $y^2 = 4ax$ meets the directrix at Q.
The line through Q parallel to the x-axis meets the normal at P at the point R.
Find the equation of the locus of R.

3

Prove by induction that

$$2^{n+2} + 3^{2n+1}$$

is divisble by 7 for all positive integers.

4

If $x = e^t$
show that

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 4y = 16 \tag{1}$$

reduces to

$$\frac{d^2 y}{dt^2} - 4y = 16$$

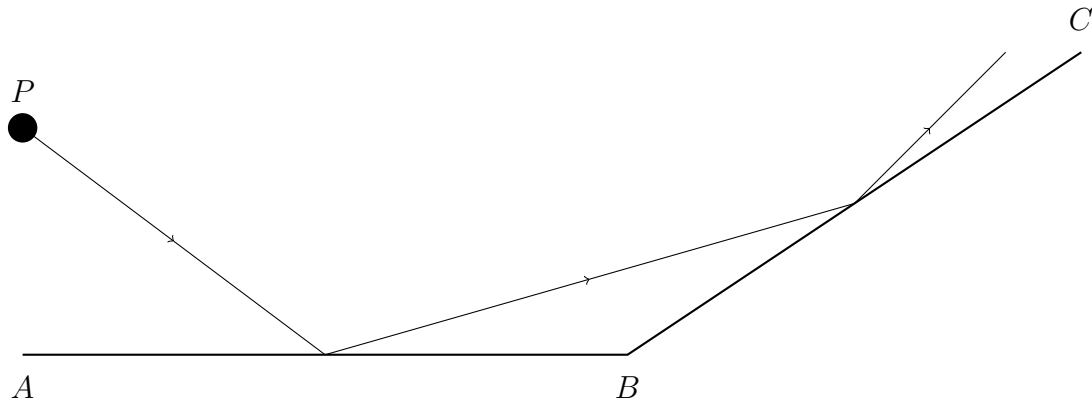Hence find the general solution for the equation (1)

5

Figure 1

Figure 1 represents the plan view of a smooth horizontal floor, where AB and BC are fixed vertical walls.

The vector $\vec{AB}$ is in the direction of $\mathbf{i}$ and the vector $\vec{BC}$ is in the direction of $(3\mathbf{i} + 2\mathbf{j})$.

A small ball P is projected across the floor towards AB. immediately before the impact with AB, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\mathrm{ms}^{-1}$.

The ball bounces off AB and then hits BC.

The ball is modelled as a particle.

The coefficient of restitution between P and AB is $\frac{1}{4}$.

The coefficient of restitution between P and BC is e.

Given that after both impacts the velocity of P is parallel to $(31\mathbf{i} + 25\mathbf{j})$ find:

(a) the value of e;

(b) the speed of P after both impacts.